Sd/- 28/4

B. Sc. Part II 4th Paper

LDE (Diff. eqns).

1. Solve $(D^2+1)y = \sin x \sin 2x$.

Soln. For CF,

$D^2+1=0 \Rightarrow D = \pm i$

$\therefore CF = c_1 e^{ix} + c_2 e^{-ix} = c_1(\cos x + i\sin x) + c_2(\cos x - i\sin x)$

$\Rightarrow CF = (c_1+c_2)\cos x + i(c_1-c_2)\sin x$

$\Rightarrow CF = A\cos x + B\sin x$

For PI

$PI = \dfrac{1}{D^2+1} \sin x \sin 2x.$

Now, $\sin x . \sin 2x = \frac{1}{2} \times 2\sin x \sin 2x$

$= \frac{1}{2}[\cos(2x-x) - \cos(2x+x)]$

$= \frac{1}{2}(\cos x - \cos 3x)$

$\Rightarrow PI = \dfrac{1}{2(D^2+1)}(\cos x - \cos 3x)$

or $PI = \dfrac{1}{2(D^2+1)}\cos x - \dfrac{1}{2(D^2+1)}\cos 3x.$ —— (1)

~~x Real ps~~

Now, $\frac{-1}{2(D^2+1)}\cos 3x = \frac{-1}{2[(-3^2)+1]}\cos 3x$

$= \frac{-1}{2\times(-8)}\cos 3x = \frac{1}{16}\cos 3x.$

Now, $\frac{1}{2(D^2+1)}\cos x$ = Real part of $\frac{1}{2(D^2+1)}e^{ix}$

$= \text{R.P. of } e^{ix}\frac{1}{2[(D+i)^2+1]}x^0$

$= \text{R.P. of } \frac{e^{ix}}{2(D^2+2Di)}x^0$

$= \text{RP. of } \frac{e^{ix}}{2\times 2Di\left(1+\frac{D^2}{2Di}\right)}x^0$

$= \text{RP. of } \frac{e^{ix}}{4Di}\left(1+\frac{D}{2i}\right)^{-1}x^0$

$= \text{RP of } \frac{e^{ix}}{4i}\times\frac{1}{D}\left(1-\frac{D}{2i}+\cdots\right)x^0$

$= \text{RP of } \frac{e^{ix}}{4i}\times\frac{1}{D}\cdot x^0 = \text{RP of } \frac{e^{ix}\,x}{4i}$

$= \text{RP of } -\frac{xi}{4}(\cos x + i\sin x) = \frac{x\sin x}{4}.$

So, $PI = \frac{x\sin x}{4} + \frac{1}{16}\cos 3x$ (using (1))

Hence, the complete soln is given by

$y = CF + PI \Rightarrow y = A\cos x + B\sin x + \frac{x\sin x}{4} + \frac{1}{16}\cos 3x.$